ARE THERE SOLITONS IN THE TWO-HIGGS STANDARD MODEL? *

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ABSTRACT

We present some evidence, based on the analysis of lower-dimensional models, for the possible existence of classically-stable winding solitons in the two-higgs electroweak theory.

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The search for stable lumps in the Weinberg-Salam model has a long history. It has revealed a rich structure of classical solutions including the sphaleron [1,2,3,4], deformed sphalerons [5,6,7,8] and vortex strings [9,10,11,12,13]. Such solutions could play a role in understanding (B+L)-violation and structure formation in the early universe, but they are all classically-unstable or/and extended. They have therefore no direct present-day manifestation, contrary to long-lived particles whose relic density could at least in principle be detected.

The existence of particle-like excitations has, on the other hand, been argued for in the context of a strongly- interacting higgs sector [14,15,16,17,18]. The advocated particles can be thought of as technibaryons of an underlying technicolor model. They are described in bosonic language by winding solitons of an effective non-renormalizable lagrangian for the pseudo-goldstone-boson (or technipion) field, much like skyrmions[19] of the effective chiral lagrangian of QCD. This is of course a phenomenological description, since the properties of such hypothetical particles cannot be calculated reliably within a semi-classical expansion. Furthermore, in view of the difficulties facing technicolor models, the possibility of a strongly-interacting higgs sector is not theoretically appealing.

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It would be clearly more interesting if classically-stable winding excitations could arise in a *weakly-coupled* scalar sector. To be more precise let us decompose the higgs-doublet field into a real (positive) magnitude and a group-phase: $\Phi = FU$, and consider static configurations with $U(x)$ wrapping N times around the $SU(2)$ manifold. These are potentially unstable for at least three distinct reasons: *(a)* because N is not conserved whenever the magnitude F goes through zero; *(b)* because N is not gauge-invariant and can, in particular, be non-vanishing even in a vacuum state; and *(c)* because scalar-field configurations can loose their energy by shrinking to zero size [20]. We refer to these for short as the *radial, gauge* and *scale* instabilities. They can be eliminated formally by *(a)* taking the physical-higgs mass $m_H \to \infty$, *(b)* decoupling the electroweak gauge fields, and *(c)* adding appropriate higher-derivative terms to the action. The question is whether classical stability can be maintained while relaxing the above conditions. This has been investigated numerically in the *minimal* case of one doublet: although one may indeed relax both the weak gauge coupling [16,5] and the higgs mass [21] up to some finite critical values, stability cannot apparently be achieved without the non-renormalizable higher-derivative terms in the action. On the other hand, as we have demonstrated recently, metastable winding solitons do arise in renormalizable models in two [22] and three [23] space-time dimensions. The way this happens is we believe instructive and could guide the search for such semi-classical solitons in four dimensions.

The simplest context in which the *radial* instability is an issue is a two-dimensional model of a complex-scalar field with mexican-hat potential: $V = \frac{1}{4}\lambda(\Phi^*\Phi - v^2)^2$. To find winding solitons we must take space to be periodic with period L †. The condition for classical stability can in this case be derived analytically and reads[22]: $m_H L > \sqrt{5}$, where $m_H = \sqrt{2\lambda}v$. The classically-relevant parameter is thus the radial-higgs mass in units of the soliton size. This follows also by comparing the loss in potential energy to the gain in gradient energy when trying to undo the winding by reducing the magnitude of the scalar. Note that the loop-expansion parameter λL^2, can be taken to zero independently so as to reach a semiclassical limit.

The above winding solitons become unstable classically if we gauge the $U(1)$ symmetry of the model. The *gauge* instability is in fact more severe than in four dimensions, because no energetic barrier opposes the turning-on of a static space-like gauge field, which is necessary to reach a winding-vacuum state. The minimal abelian-higgs model has thus only unstable (sphaleron) solutions [25] ‡ . The situation changes, however, drastically if there are more than one complex scalars. The gauge-invariant relative phases of any two of them cannot in this case wind around non-trivially in a vacuum state. An explicit analysis of this extended abelian-higgs model [22] shows that winding solitons persist down to scalar masses close to the inverse soliton size: gauging and the extra higgs enhance the stability region found in the global model.

† Alternatively we may add a mass term: $\delta V = -\mu^2 v Re(\Phi)$, that lifts the vacuum degeneracy. Stable winding excitations, which reduce to the sine-Gordon solitons in the $\lambda \to \infty$ limit, can be shown [24] numerically to exist for $\lambda v^2/\mu > 18.8$.

‡ It was claimed erroneously in [22] that it has no static solutions whatsoever. This is only correct in the $\lambda \to \infty$ limit.

The *scale* instability becomes an issue for the first time in three space-time dimensions. To be more precise we consider a real-triplet scalar field $\Phi_a(x)$ $(a = 1, 2, 3)$ with mexican-hat potential : $V = \frac{1}{4}\lambda(\Phi_a\Phi_a - v^2)^2$. The limit $\lambda \to \infty$ corresponds to the $O(3)$ non-linear σ-model. This is known to possess winding solitons, characterized by non-trivial mappings of the two-sphere onto itself, and having arbitrary size [26]. For finite λ on the other hand, or in the presence of a symmetry-breaking potential, Derrick's scaling argument [20] shows that these solitons are unstable to shrinking. One can of course again invoke higher-derivative terms to stabilize the scale[27]. The same result is however in this case achieved by a massive $U(1)$ gauge field with only renormalizable couplings [23]. This can be established by perturbing around the $O(3)$ non-linear σ-model limit, or else by solving numerically the equations of motion.

What do these lower-dimensional solitons teach us? First, they are interesting in their own right, since they correspond to a new class of wall and string defects in renormalizable four-dimensional models. Second, they suggest by analogy that classically-stable winding solitons may exist in a weakly-coupled two-higgs extension of the standard model. These hypothetical solitons would: *(b)* be characterized by the non-trivial winding of the relative phase of the two doublets, and thus be immune to the gauge mode of decay; *(c)* have a scale stabilized by electroweak magnetic fields and hence of order $1/m_W$; and *(a)* hopefully stay stable for higgs masses near m_W and thus compatible with perturbative unitarity [§]. Mathematically the situation is the same as in the hidden-gauge-boson models [28,29] of strong and electroweak interactions, except that the role of the hidden gauge bosons is here played by $W^{\pm}$ and Z themselves. Although previous numerical investigations [28,30] have shown no sign of stable solitons in these contexts, a systematic search is in our opinion necessary in order to settle definitely the issue [31].

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[§] Though admittedly premature, some other physical properties of such would-be particles are fun to contemplate: being classically stable they could easily have cosmological life times. They would have a mass in the $\sim 10\ TeV$ region, zero charge and dipole moments in their ground state, and geometrical interaction cross sections of order $1/m_W^2$. Assuming maximum production at the electroweak phase transition, a rough estimate of their present abundance shows that they could be candidates for cold dark matter in the universe.

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